

Mukavemet - özümlü Örnekler

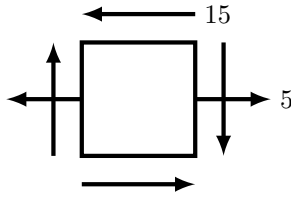
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Bölüm 1

Düzlem Gerilme

1.1 Örnek



Şekilde görülen gerilme hâlinde;

$$\begin{aligned}\sigma_x &= 5 \text{ N/mm}^2 \\ \sigma_y &= 0 \text{ N/mm}^2 \\ \tau_{xy} &= -15 \text{ N/mm}^2 \text{ dir.}\end{aligned}$$

- Normali yatay eksenle 30° 'lik açı yapan bir yüzeydeki σ ve τ gerilmelerini;
- Asal normal gerilmeleri ve doğrultularını;
- Asal kayma gerilmelerini hesaplayınız.

a. Normali yatay eksenle 30° 'lik açı yapan bir yüzeydeki σ ve τ gerilmeleri :

σ değerinin hesaplanması:

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\varphi + \tau_{xy} \cdot \sin 2\varphi$$

$$\sigma = \frac{5 + 0}{2} + \frac{5 - 0}{2} \cdot \cos 60 - 15 \cdot \sin 60$$

$$\sigma = 2.5 + 1.25 - 12.99$$

$$\underline{\sigma = -9.24 \text{ N/mm}^2}$$

τ değerinin hesaplanması:

$$\tau = -\frac{\sigma_x - \sigma_y}{2} \cdot \sin 2\varphi + \tau_{xy} \cdot \cos 2\varphi$$

$$\tau = -\frac{5 - 0}{2} \cdot \sin 60 - 15 \cdot \cos 60$$

$$\tau = -2.17 - 7.5$$

$$\underline{\tau = -9.67 \text{ N/mm}^2}$$

b. Asal normal gerilmeler ve doğrultuları :

σ_1 ve σ_2 değerlerinin hesaplanması:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{5 + 0}{2} \pm \sqrt{\left(\frac{5 - 0}{2}\right)^2 + (-15)^2}$$

$$\sigma_{1,2} = 2.5 \pm 15.21$$

$$\underline{\sigma_1 = 17.71 \text{ N/mm}^2}$$

$$\underline{\sigma_2 = -12.71 \text{ N/mm}^2}$$

φ_0 değerinin hesaplanması:

$$\operatorname{tg} 2\varphi_0 = \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}$$

$$\operatorname{tg} 2\varphi_0 = \frac{2 \cdot (-15)}{5 - 0}$$

$$\operatorname{tg} 2\varphi_0 = \frac{-30}{5}$$

$$\operatorname{tg} 2\varphi_0 = -6$$

$$\underline{\varphi_0 = -40.3^\circ}$$

$$\underline{\varphi_0 = 49.7^\circ}$$

c. Asal kayma gerilmeleri : τ_{max} ve τ_{min} değerlerinin hesaplanması:

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

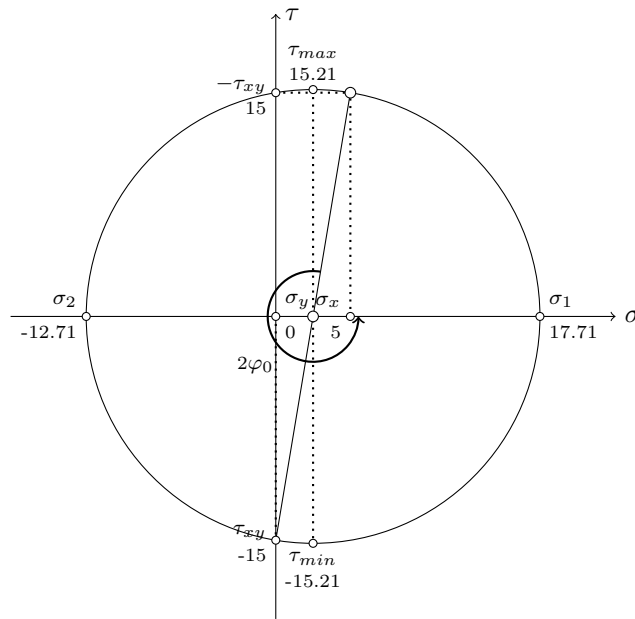
$$\tau_{\max/\min} = \pm \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{5 - 0}{2}\right)^2 - 15^2}$$

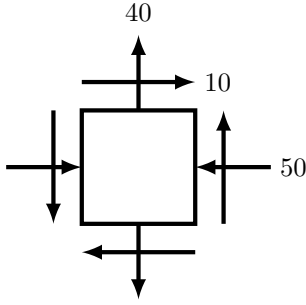
$$\tau_{\max/\min} = \pm \frac{17.71 + 12.71}{2}$$

$$\tau_{\max/\min} = \pm 15.21 \text{ N/mm}^2$$

$$\tau_{\max/\min} = \pm 15.21 \text{ N/mm}^2$$



1.2 Örnek



Şekilde görülen gerilme hâlinde;

$$\begin{aligned}\sigma_x &= -50 \text{ N/mm}^2 \\ \sigma_y &= 40 \text{ N/mm}^2 \\ \tau_{xy} &= 10 \text{ N/mm}^2 \text{ dir.}\end{aligned}$$

- Normali yatay eksenle 30° 'lik açı yapan bir yüzeydeki σ ve τ gerilmelerini;
- Asal normal gerilmeleri ve doğrultularını;
- Asal kayma gerilmelerini hesaplayınız.

a. Normali yatay eksenle 30° 'lik açı yapan bir yüzeydeki σ ve τ gerilmeleri :

σ değerinin hesaplanması:

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\varphi + \tau_{xy} \cdot \sin 2\varphi$$

$$\sigma = \frac{-50 + 40}{2} + \frac{-50 - 40}{2} \cdot \cos 60 + 10 \cdot \sin 60$$

$$\sigma = -5 - 22.5 + 8.66$$

$$\underline{\sigma = -18.84 \text{ N/mm}^2}$$

τ değerinin hesaplanması:

$$\tau = -\frac{\sigma_x - \sigma_y}{2} \cdot \sin 2\varphi + \tau_{xy} \cdot \cos 2\varphi$$

$$\tau = -\frac{-50 - 40}{2} \cdot \sin 60 + 10 \cdot \cos 60$$

$$\tau = 38.97 + 5$$

$$\underline{\tau = 43.97 \text{ N/mm}^2}$$

b. Asal normal gerilmeler ve doğrultuları :

σ_1 ve σ_2 değerlerinin hesaplanması:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-50 + 40}{2} \pm \sqrt{\left(\frac{-50 - 40}{2}\right)^2 + 10^2}$$

$$\sigma_{1,2} = -5 \pm 46.1$$

$$\underline{\sigma_1 = 41.1 \text{ N/mm}^2}$$

$$\underline{\sigma_2 = -51.1 \text{ N/mm}^2}$$

φ_0 değerinin hesaplanması:

$$\operatorname{tg} 2\varphi_0 = \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}$$

$$\operatorname{tg} 2\varphi_0 = \frac{2 \cdot 10}{-50 - 40}$$

$$\operatorname{tg} 2\varphi_0 = \frac{20}{-90}$$

$$\operatorname{tg} 2\varphi_0 = -0.2222$$

$$\underline{\varphi_0 = -6.3^\circ}$$

$$\underline{\varphi_0 = 83.7^\circ}$$

c. Asal kayma gerilmeleri :

 τ_{max} ve τ_{min} değerlerinin hesaplanması:

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

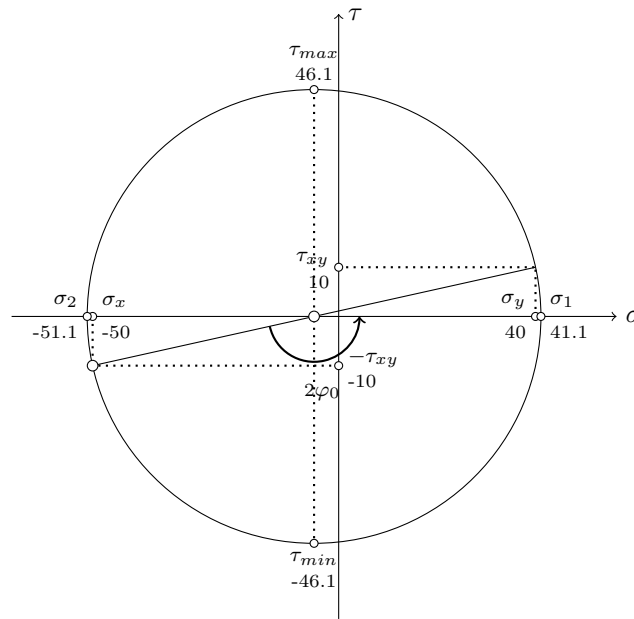
$$\tau_{\max/\min} = \pm \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{-50 - 40}{2}\right)^2 + 10^2}$$

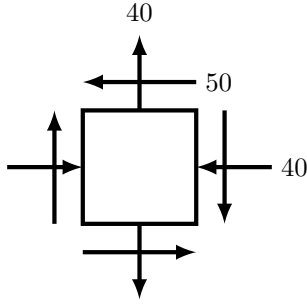
$$\tau_{\max/\min} = \pm \frac{41.1 + 51.1}{2}$$

$$\tau_{\max/\min} = \pm 46.1 \text{ N/mm}^2$$

$$\tau_{\max/\min} = \pm 46.1 \text{ N/mm}^2$$



1.3 Örnek



Şekilde görülen gerilme hâlinde;

$$\begin{aligned}\sigma_x &= -40 \text{ N/mm}^2 \\ \sigma_y &= 40 \text{ N/mm}^2 \\ \tau_{xy} &= -50 \text{ N/mm}^2, \text{ dir.}\end{aligned}$$

- Normali yatay eksenle -30° 'lik açı yapan bir yüzeydeki σ ve τ gerilmelerini;
- Asal normal gerilmeleri ve doğrultularını;
- Asal kayma gerilmelerini hesaplayınız.

a. Normali yatay eksenle -30° 'lik açı yapan bir yüzeydeki σ ve τ gerilmeleri :

σ değerinin hesaplanması:

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\varphi + \tau_{xy} \cdot \sin 2\varphi$$

$$\sigma = \frac{-40 + 40}{2} + \frac{-40 - 40}{2} \cdot \cos(-60) - 50 \cdot \sin(-60)$$

$$\sigma = 0 - 20 + 43.3$$

$$\underline{\sigma = 23.3 \text{ N/mm}^2}$$

τ değerinin hesaplanması:

$$\tau = -\frac{\sigma_x - \sigma_y}{2} \cdot \sin 2\varphi + \tau_{xy} \cdot \cos 2\varphi$$

$$\tau = -\frac{-40 - 40}{2} \cdot \sin(-60) - 50 \cdot \cos(-60)$$

$$\tau = -34.64 - 25$$

$$\underline{\tau = -59.64 \text{ N/mm}^2}$$

b. Asal normal gerilmeler ve doğrultuları :

σ_1 ve σ_2 değerlerinin hesaplanması:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-40 + 40}{2} \pm \sqrt{\left(\frac{-40 - 40}{2}\right)^2 + (-50)^2}$$

$$\sigma_{1,2} = 0 \pm 64.03$$

$$\underline{\sigma_1 = 64.03 \text{ N/mm}^2}$$

$$\underline{\sigma_2 = -64.03 \text{ N/mm}^2}$$

φ_0 değerinin hesaplanması:

$$\text{tg}2\varphi_0 = \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}$$

$$\text{tg}2\varphi_0 = \frac{2 \cdot (-50)}{-40 - 40}$$

$$\text{tg}2\varphi_0 = \frac{-100}{-80}$$

$$\text{tg}2\varphi_0 = 1.25$$

$$\underline{\varphi_0 = 25.7^\circ}$$

$$\underline{\varphi_0 = 115.7^\circ}$$

c. Asal kayma gerilmeleri : τ_{max} ve τ_{min} değerlerinin hesaplanması:

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

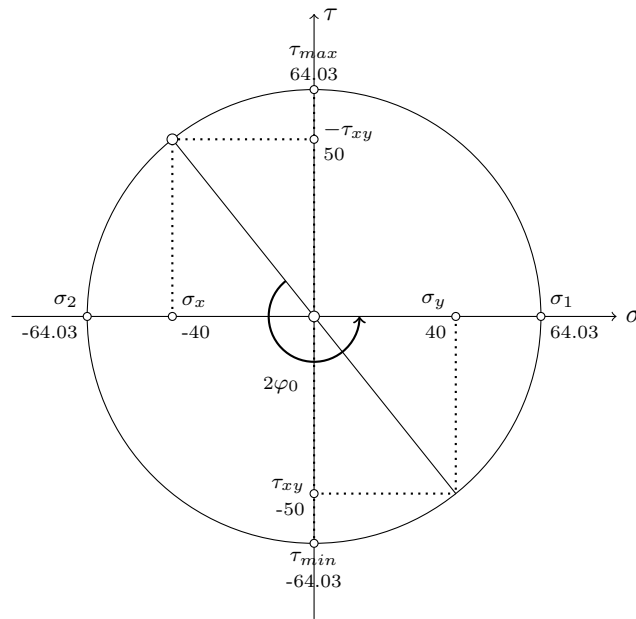
$$\tau_{\max/\min} = \pm \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{-40 - 40}{2}\right)^2 - 50^2}$$

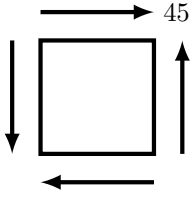
$$\tau_{\max/\min} = \pm \frac{64.03 + 64.03}{2}$$

$$\tau_{\max/\min} = \pm 64.03 \text{ N/mm}^2$$

$$\tau_{\max/\min} = \pm 64.03 \text{ N/mm}^2$$



1.4 Örnek



Şekilde görülen gerilme hâlinde;

$$\begin{aligned}\sigma_x &= 0 \text{ N/mm}^2 \\ \sigma_y &= 0 \text{ N/mm}^2 \\ \tau_{xy} &= 45 \text{ N/mm}^2 \text{ dir.}\end{aligned}$$

- Normali yatay eksenle 30° 'lik açı yapan bir yüzeydeki σ ve τ gerilmelerini;
- Asal normal gerilmeleri ve doğrultularını;
- Asal kayma gerilmelerini hesaplayınız.

a. Normali yatay eksenle 30° 'lik açı yapan bir yüzeydeki σ ve τ gerilmeleri :

σ değerinin hesaplanması:

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\varphi + \tau_{xy} \cdot \sin 2\varphi$$

$$\sigma = \frac{0 + 0}{2} + \frac{0 - 0}{2} \cdot \cos 60 + 45 \cdot \sin 60$$

$$\sigma = 0 + 0 + 38.97$$

$$\underline{\sigma = 38.97 \text{ N/mm}^2}$$

τ değerinin hesaplanması:

$$\tau = -\frac{\sigma_x - \sigma_y}{2} \cdot \sin 2\varphi + \tau_{xy} \cdot \cos 2\varphi$$

$$\tau = -\frac{0 - 0}{2} \cdot \sin 60 + 45 \cdot \cos 60$$

$$\tau = 0 + 22.5$$

$$\underline{\tau = 22.5 \text{ N/mm}^2}$$

b. Asal normal gerilmeler ve doğrultuları :

σ_1 ve σ_2 değerlerinin hesaplanması:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{0 + 0}{2} \pm \sqrt{\left(\frac{0 - 0}{2}\right)^2 + 45^2}$$

$$\sigma_{1,2} = 0 \pm 45$$

$$\underline{\sigma_1 = 45 \text{ N/mm}^2}$$

$$\underline{\sigma_2 = -45 \text{ N/mm}^2}$$

φ_0 değerinin hesaplanması:

$$\text{tg}2\varphi_0 = \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}$$

$$\text{tg}2\varphi_0 = \frac{2 \cdot 45}{0 - 0}$$

$$\text{tg}2\varphi_0 = \frac{90}{0}$$

$$\text{tg}2\varphi_0 = \infty$$

$$\underline{\varphi_0 = 45^\circ}$$

$$\underline{\varphi_0 = 135^\circ}$$

c. Asal kayma gerilmeleri : τ_{max} ve τ_{min} değerlerinin hesaplanması:

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

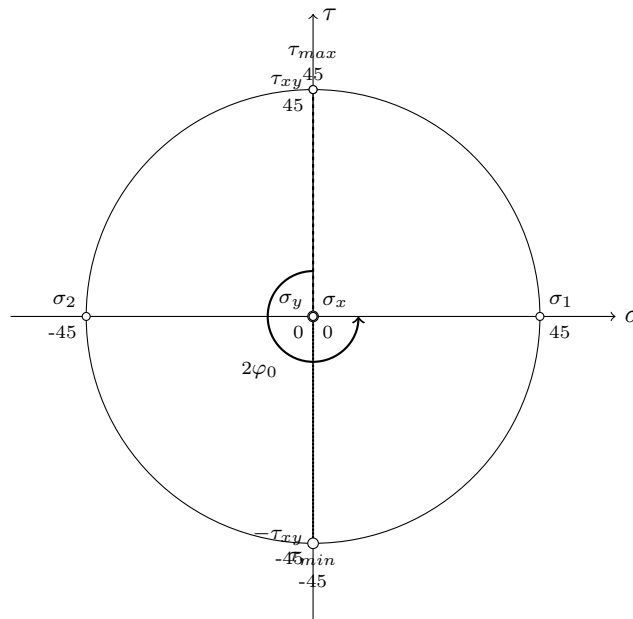
$$\tau_{\max/\min} = \pm \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{0 - 0}{2}\right)^2 + 45^2}$$

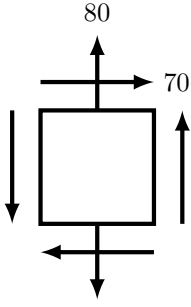
$$\tau_{\max/\min} = \pm \frac{45 + 45}{2}$$

$$\tau_{\max/\min} = \pm 45 \text{ N/mm}^2$$

$$\tau_{\max/\min} = \pm 45 \text{ N/mm}^2$$



1.5 Örnek



Şekilde görülen gerilme hâlinde;

$$\begin{aligned}\sigma_x &= 0 \text{ N/mm}^2 \\ \sigma_y &= 80 \text{ N/mm}^2 \\ \tau_{xy} &= 70 \text{ N/mm}^2 \text{ dir.}\end{aligned}$$

- Normali yatay eksenle 45° 'lik açı yapan bir yüzeydeki σ ve τ gerilmelerini;
- Asal normal gerilmeleri ve doğrultularını;
- Asal kayma gerilmelerini hesaplayınız.

a. Normali yatay eksenle 45° 'lik açı yapan bir yüzeydeki σ ve τ gerilmeleri :

σ değerinin hesaplanması:

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\varphi + \tau_{xy} \cdot \sin 2\varphi$$

$$\sigma = \frac{0 + 80}{2} + \frac{0 - 80}{2} \cdot \cos 90 + 70 \cdot \sin 90$$

$$\sigma = 40 + 0 + 70$$

$$\underline{\sigma = 110 \text{ N/mm}^2}$$

τ değerinin hesaplanması:

$$\tau = -\frac{\sigma_x - \sigma_y}{2} \cdot \sin 2\varphi + \tau_{xy} \cdot \cos 2\varphi$$

$$\tau = -\frac{0 - 80}{2} \cdot \sin 90 + 70 \cdot \cos 90$$

$$\tau = 40 + 0$$

$$\underline{\tau = 40 \text{ N/mm}^2}$$

b. Asal normal gerilmeler ve doğrultuları :

σ_1 ve σ_2 değerlerinin hesaplanması:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{0 + 80}{2} \pm \sqrt{\left(\frac{0 - 80}{2}\right)^2 + 70^2}$$

$$\sigma_{1,2} = 40 \pm 80.62$$

$$\underline{\sigma_1 = 120.62 \text{ N/mm}^2}$$

$$\underline{\sigma_2 = -40.62 \text{ N/mm}^2}$$

φ_0 değerinin hesaplanması:

$$\operatorname{tg} 2\varphi_0 = \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}$$

$$\operatorname{tg} 2\varphi_0 = \frac{2 \cdot 70}{0 - 80}$$

$$\operatorname{tg} 2\varphi_0 = \frac{140}{-80}$$

$$\operatorname{tg} 2\varphi_0 = -1.75$$

$$\underline{\varphi_0 = -30.1^\circ}$$

$$\underline{\varphi_0 = 59.9^\circ}$$

c. Asal kayma gerilmeleri : τ_{max} ve τ_{min} değerlerinin hesaplanması:

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

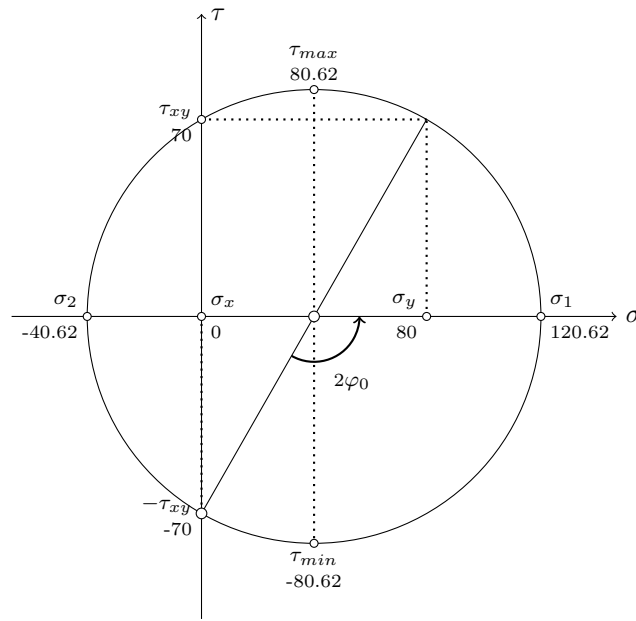
$$\tau_{\max/\min} = \pm \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{0 - 80}{2}\right)^2 + 70^2}$$

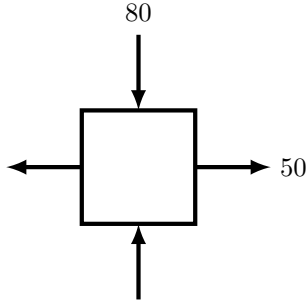
$$\tau_{\max/\min} = \pm \frac{120.62 + 40.62}{2}$$

$$\tau_{\max/\min} = \pm 80.62 \text{ N/mm}^2$$

$$\tau_{\max/\min} = \pm 80.62 \text{ N/mm}^2$$



1.6 Örnek



Şekilde görülen gerilme hâlinde;

$$\begin{aligned}\sigma_x &= 50 \text{ N/mm}^2 \\ \sigma_y &= -80 \text{ N/mm}^2 \\ \tau_{xy} &= 0 \text{ N/mm}^2 \text{ dir.}\end{aligned}$$

- Normali yatay eksenle 35° 'lik açı yapan bir yüzeydeki σ ve τ gerilmelerini;
- Asal normal gerilmeleri ve doğrultularını;
- Asal kayma gerilmelerini hesaplayınız.

a. Normali yatay eksenle 35° 'lik açı yapan bir yüzeydeki σ ve τ gerilmeleri :

σ değerinin hesaplanması:

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cdot \cos 2\varphi + \tau_{xy} \cdot \sin 2\varphi$$

$$\sigma = \frac{50 - 80}{2} + \frac{50 + 80}{2} \cdot \cos 70 + 0 \cdot \sin 70$$

$$\sigma = -15 + 22.23 + 0$$

$$\underline{\sigma = 7.23 \text{ N/mm}^2}$$

τ değerinin hesaplanması:

$$\tau = -\frac{\sigma_x - \sigma_y}{2} \cdot \sin 2\varphi + \tau_{xy} \cdot \cos 2\varphi$$

$$\tau = -\frac{50 + 80}{2} \cdot \sin 70 + 0 \cdot \cos 70$$

$$\tau = -61.08 + 0$$

$$\underline{\tau = -61.08 \text{ N/mm}^2}$$

b. Asal normal gerilmeler ve doğrultuları :

σ_1 ve σ_2 değerlerinin hesaplanması:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{50 - 80}{2} \pm \sqrt{\left(\frac{50 + 80}{2}\right)^2 + 0^2}$$

$$\sigma_{1,2} = -15 \pm 65$$

$$\underline{\sigma_1 = 50 \text{ N/mm}^2}$$

$$\underline{\sigma_2 = -80 \text{ N/mm}^2}$$

φ_0 değerinin hesaplanması:

$$\text{tg}2\varphi_0 = \frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}$$

$$\text{tg}2\varphi_0 = \frac{2 \cdot 0}{50 + 80}$$

$$\text{tg}2\varphi_0 = \frac{0}{130}$$

$$\text{tg}2\varphi_0 = 0$$

$$\underline{\varphi_0 = 0^\circ}$$

$$\underline{\varphi_0 = 90^\circ}$$

c. Asal kayma gerilmeleri : τ_{max} ve τ_{min} değerlerinin hesaplanması:

$$\tau_{\min}^{\max} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\min}^{\max} = \pm \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{\min}^{\max} = \pm \sqrt{\left(\frac{50 + 80}{2}\right)^2 + 0^2}$$

$$\tau_{\min}^{\max} = \pm \frac{50 + 80}{2}$$

$$\tau_{\min}^{\max} = \pm 65 \text{ N/mm}^2$$

$$\tau_{\min}^{\max} = \pm 65 \text{ N/mm}^2$$

