



Total Potential Energy Optimization Method Analyses of Trusses Structures Considering the Elasto-Plastic Behavior

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Abstract

The nonlinear behavior of the structure is an important subject that must be taken into consideration during the structural design. This behavior can occur due to large deformations and/or material nonlinearity. Elasto-plastic behavior is a well-known idealization that makes the structure highly nonlinear. By using the method of Total Potential Optimization using Meta-heuristic Algorithms (TPO/MA), this behavior can be easily taken into consideration, whether the deformations are large or not. According to total potential energy theory, the structure is in equilibrium when the energy of the structure is minimum. The idea of using metaheuristic algorithms for this minimization process is resulted in the TPO/MA method. In this study, the TPO/MA method of analyses of truss structures are presented considering the elasto-plastic behavior. As a metaheuristic algorithm, the harmony search (HS) algorithm is employed. The analyses presented show that the technique is feasible, robust and effective in solving these kinds of problems.

Keywords: *Truss, Elasto-plastic behavior, Total Potential Optimization using Meta-heuristic Algorithms (TPO/MA), Harmony search, nonlinear structure.*

1 Introduction

The main aim of structural analysis is to determine internal forces, i.e. flexural moment, torsional moment, shear forces, axial forces, of structural members under the effect of external loading such as earthquake, wind, dead and live loads etc. In design process, after determining the internal forces, each member is designed by considering two main aspects, safety measures and economic conditions. These tasks can be provided by taking into consideration structural and material nonlinear behaviour.

In the classical analyses techniques, the linear analyses are conducting by using $P=K\Delta$, where P is load vector, K is stiffness matrix (square matrix that consists from geometric and material properties of the members, i.e. moment of inertia, elasticity modulus, length of the member) and Δ is displacement vector, and by applying matrix operations. These matrices and vectors are derivate from the equilibrium conditions of the system. However for linear problem creating these matrices is easy task, for the nonlinear problem (because of force-

displacement behaviour is not linearly) is hard and often impossible. In these approaches iterative analyses are conducting in order to overcome this problem.

The method called total potential energy optimization using meta-heuristic algorithms (TPO/MA) (Toklu 2004; Toklu et al. 2013; Toklu and Toklu 2013) can be used in the analyses instead of matrix operation methods. The total potential energy principle is a well-known principle in mechanics. According to this principle, the total potential energy of the system is minimum when the system in equilibrium. In this situation, TPO/MA method employs the metaheuristic algorithms, developed for minimization or maximization problems, for finding the minimum total potential energy of the system. Using this approach brings an advantage that makes the TPO/MA method effective for linear or nonlinear behavior.

In this paper, truss structure analyses are presented by using TPO/MA method. The analyses are done considering nonlinear material behaviour. As metaheuristic algorithm harmony search is employed. Analyses results show that, the TPO/MA method is effective and easily applied method for nonlinear behaviour of the material.

2 Harmony Search and Methodology

Metaheuristic algorithms are developed based on natural phenomena such as Genetic algorithm from natural selection (Holland, 1975 and Goldberg, 1989), simulated annealing from annealing process in metallurgy (Kirkpatrick et al.,1983), particle swarm from movement of organisms bird flock or fish school (Kennedy and Eberhart,1995), big bang-big crunch from evolution of universe (Erol and Eksin, 2006), etc. Harmony search, one of the metaheuristic algorithms, is developed from inspiration of the musical performance of musician.

Total energy minimization process via harmony search can be summarised in five steps.

Step I: In the first step, the data of the optimization problem is reading from a pre-prepared file. This file contains design constrains; material properties, cross-sectional dimensions of members, geometrical properties of system, boundary condition of joints, external loads and special parameters of HS algorithm; harmony memory size (HMS), harmony memory considering rate (HMCR) and pitch adjacent rate (PAR).

Step II: In the second step, initial harmony memory (HM) matrix is constructed by using harmony vectors (HVs) as many as HMCR (p). In each HV, the randomly generated displacements and potential energy of system that calculated for these displacements is stored.

Step III: The best (the HV in HM matrix with minimum energy) and the worst (the HV in HM matrix with maximum energy) vectors are determined according to total potential energy values calculated in Step II.

Step IV: A new HV is generated by using special rules of HS algorithm. According to HS rules, this vector can be generated from whole solution range as did initial HVs or from close range (PAR times of the whole solution range) around one of HV stored in HM. After calculation the energy of the new vector, it is added to HM matrix and then the worst vector in matrix is deleted.

Step V: In fifth and the last step, the stopping criterion is checked. This criterion is maximum iteration number and it is defined by user. If this criterion is not satisfied, respectively operation described in Step IV, III and V is repeated. At the end of the optimization the optimum results is outputted (for detail info of the process see: Toklu et al. (2013)).

3 Numerical Examples

Example 1: 6-bar truss

The first example is a 6-bar truss system (Fig. 1). Cross-sectional dimension of members are 100 mm^2 and $P=150 \text{ kN}$ concentrated load at node 4. The analyses were performed for three material properties given in Fig. 2. As seen in figure one of the materials is defined as linear elastic.

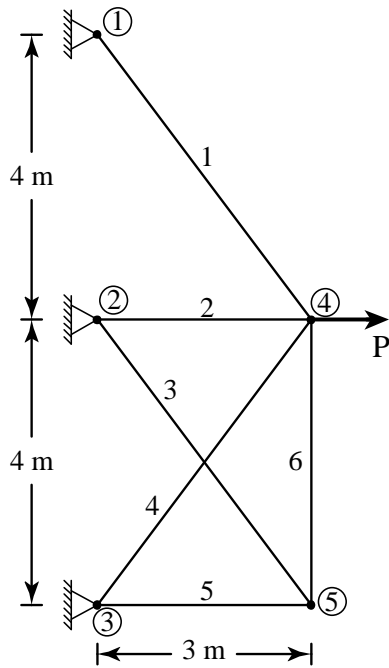


Figure 1. 6-Member truss system with $P=150$ kN load

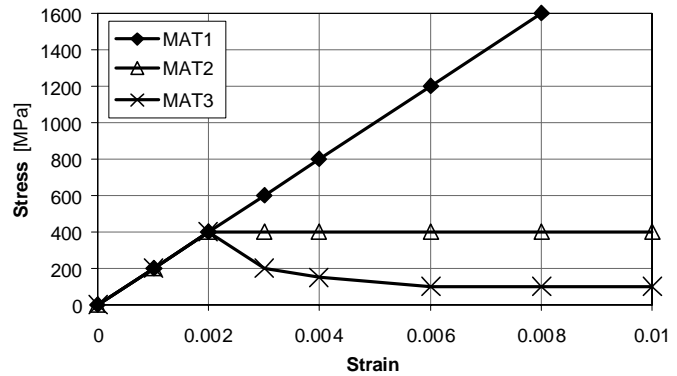


Figure 2. Stress–strain diagrams of materials

The deformed shape of the system for minimum potential energy (equilibrium state for defined load) can be seen in Fig.3. As it is expected, the minimum and maximum deformation are found for MAT1 and MAT3, respectively. Also, potential energy, joint displacement at node 4 (for $x(u_4)$ and $y(v_4)$), joint displacement at node 5 (for $x(u_5)$ and $y(v_5)$) and member forces are given for presented method TPO/MA (using HS algorithm), for linear and nonlinear analyses of finite element method (FEM) and for Toklu (2004) in Table 1. As seen in table, except for the FEM linear solution, all results seems compatible. Some other graphs of optimization process is given in Figs. 4-9.

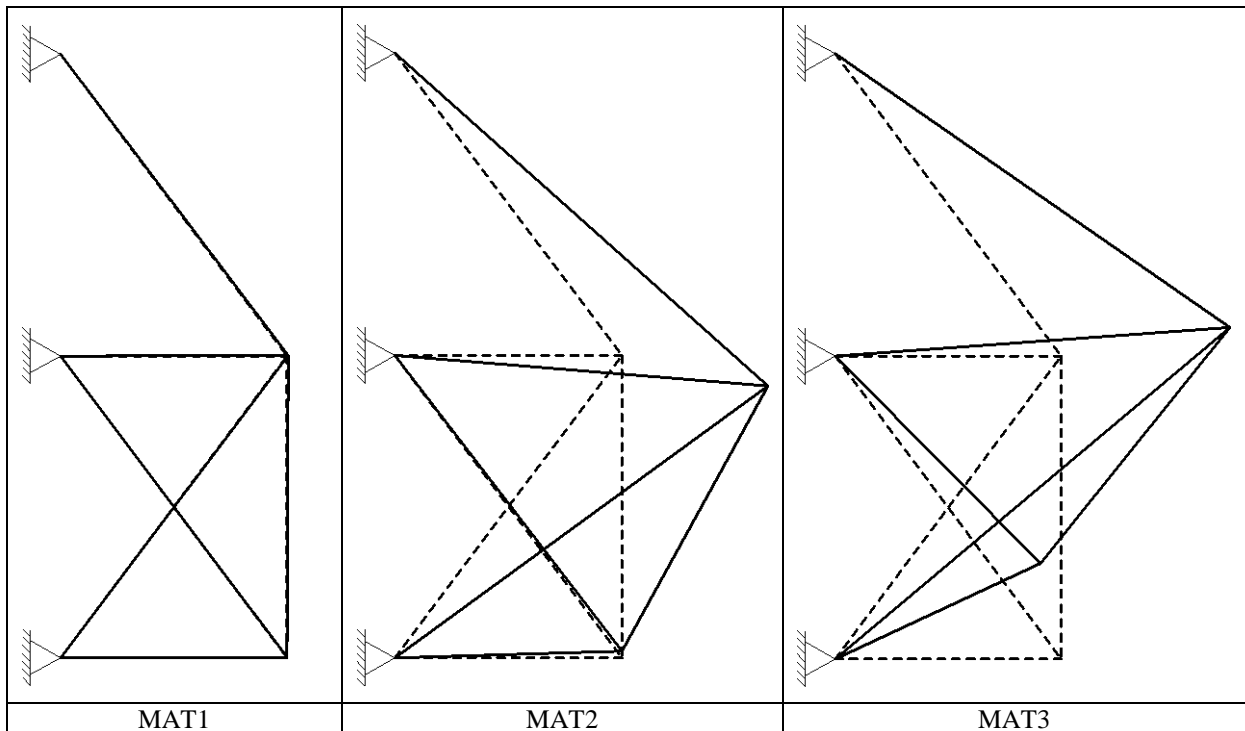


Figure 3. Deformed shape of the system for minimum potential energies.

Table 1. Joint displacements and member forces for 6-member truss with different materials and different solution techniques

Material	Energy [kJNm]	Joint Displacements [mm]				Member Forces [kN]					
	U	$u(4)$	$v(4)$	$u(5)$	$v(5)$	$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
MAT1 ^a	-1.059727	14.15	2.84	0.30	2.31	49.725	94.331	-6.669	43.056	4.001	5.335
MAT1 ^b	-1.059735	14.12	2.83	0.30	2.32	49.810	94.14	-6.688	42.973	4.034	5.351
MAT1 ^c	-1.059735	14.12	2.83	0.30	2.32	49.811	94.142	-6.688	42.974	4.035	5.352
MAT1 ^d	-1.059735	14.12	2.82	0.30	2.31	49.863	94.143	-6.671	42.949	4.013	5.339
MAT2 ^c	-36.75831	1931.60	-403.79	1.89	88.17	80.000	40.000	-40.000	40.000	42.512	37.642
MAT2 ^d	-36.75835	1931.00	-402.64	1.84	89.00	80.000	40.000	-40.000	40.000	42.122	37.793
MAT3 ^c	-52.09811	2235.51	387.11	-283.55	1277.18	80.000	40.000	-10.000	40.000	22.839	21.616
MAT3 ^d	-52.09777	2234.42	383.46	-280.13	1269.86	80.000	40.000	-10.000	40.000	22.744	21.826

^a Linear solution obtained using FEM

^b Geometrically nonlinear solution obtained using FEM

^c Toklu (2004)

^d Present Method

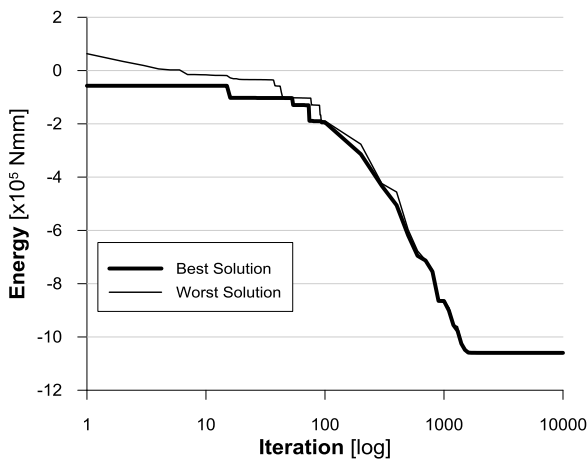


Figure 4. Energy vs iteration number for the best and worst solutions for MAT1^d

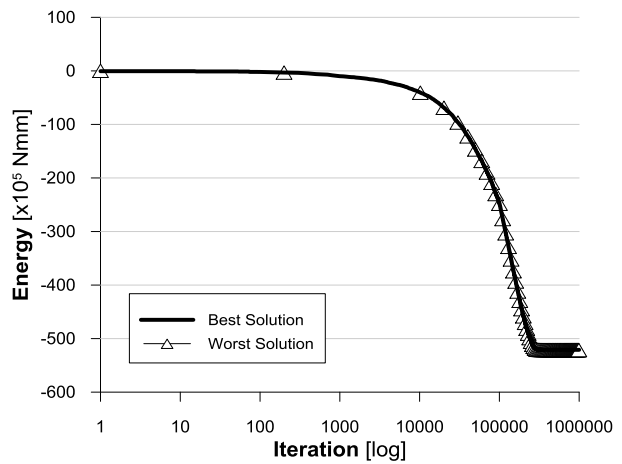


Figure 5. Energy vs iteration number for the best and worst solutions for MAT3^d

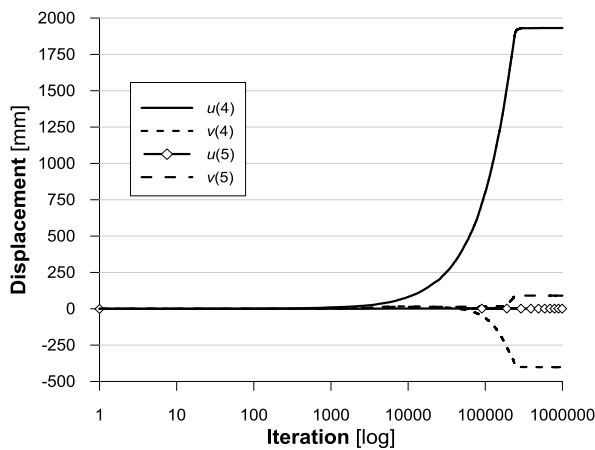


Figure 6. Displacement vs iteration number of joints 4 and 5 (MAT2)

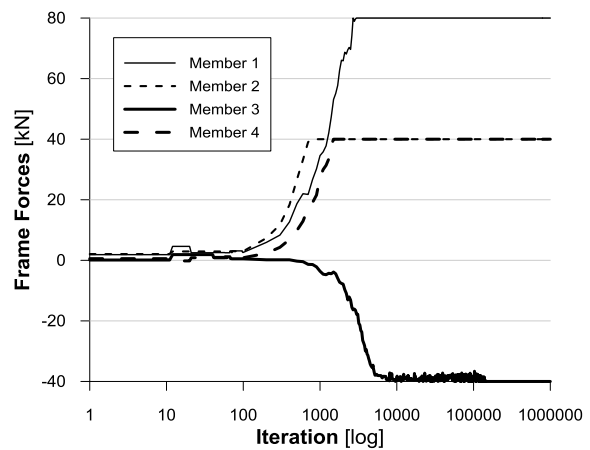


Figure 7. Member forces vs iteration number of members 1-4 (MAT2)

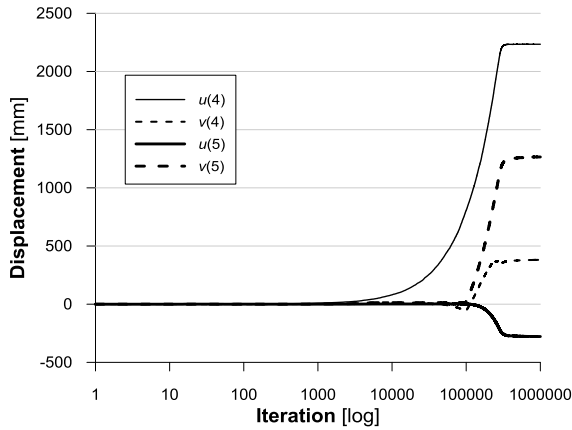


Figure 8. Displacement vs iteration number of joints 4 and 5 (MAT3)

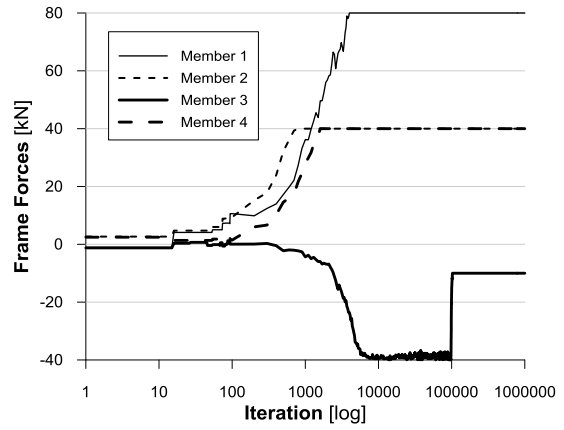


Figure 9. Member forces vs iteration number of members 1-4 (MAT3)

Example 1: 3-bar truss

In the second example, presented method is performed on a 3-bar truss system (Fig. 10) with 15500 mm² cross-sectional dimension of members. The material is elastic-perfectly plastic with yield strength and elasticity modulus of the material are 250 MPa and 200000 N/mm², respectively.

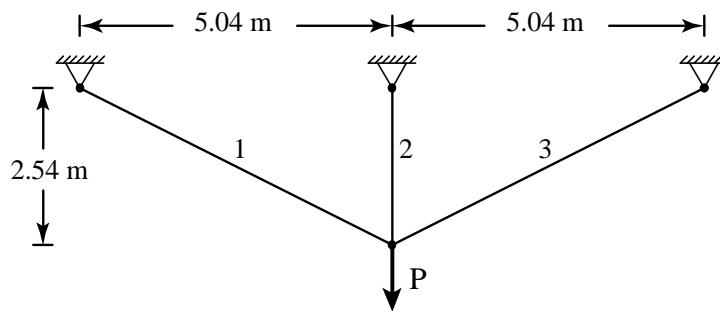


Figure 10. 3-bar truss system

In Figure 11, load vs vertical displacement values can be seen. Also, yield load for each members obtained from presented method and other two documented method is given in Table 2. As seen in table, the results are highly compatible with each other and the best solution (closest yield strength) is obtained from presented method.

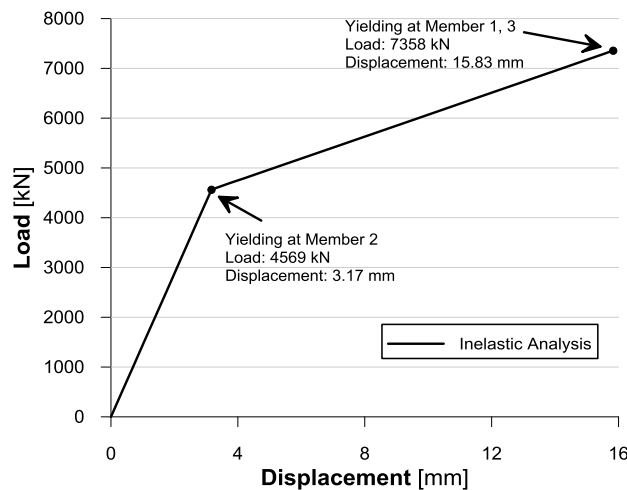


Figure 11. Load-Vertical Displacement of Three-Element Truss

Table 2. Members yielding loads of 3-bar truss

	Kim et al. (2001)		Seçer (2009)		Present Method	
	Yielding Member 2	Yielding at Member 1, 3	Yielding Member 2	Yielding Member 1, 3	Yielding Member 2	Yielding Member 1, 3
Load [kN]	4557	7365	4563	7339	4569	7358
Stress* [MPa]	249.33	250.49	249.66	248.63	249.99	249.99

* Obtained using FEM

4 Conclusions

In the study, truss structures analyses are investigated by taken into consideration geometrical en material nonlinear behaviour. The analyses were performed by using TPO/MA (using HS algorithm) method, developed based on idea of using meta-heuristic algorithm for the minimization of well-known mechanic principle called total potential energy. Two examples were presented in the study. It is seen that form these applications of proposed method; method is effective and easily applicable for nonlinear analyses of truss structures.

References

- Erol, O.K., Eksin, I. (2006). A new optimization method: Big bang big crunch. *Advances in Engineering Software*, Vol. 37, pp. 106-111.
- Geem, Z.W., Kim J.H., Loganathan, G.V. (2001). A new heuristic optimization algorithm: harmony search. *Simul.*, Vol. 76, pp. 60–68.
- Goldberg, D.E. (1989). *Genetic algorithms in search, Optimization and machine learning*. Addison Wesley, Boston, Massachusetts.
- Holland, J.H. (1975). *Adaptation in Natural and Artificial Systems*. University of Michigan Press, Ann Arbor, Michigan.
- Kennedy, J. and Eberhart, R.C. (1995). Particle swarm optimization. In: *Proceedings of IEEE International Conference on Neural Networks No. IV*, November 27-December 1, pp. 1942–1948, Perth Australia.
- Kim, S. E., Park, M. H. and Choi, S. H. (2001). Practical advanced analysis and design of three-dimensional truss bridges. *Journal of Constructional Steel Research*, Vol. 57(8), pp. 907-923.
- Kirkpatrick, S., Gelatt, C. and Vecchi, M. (1983). Optimization by simulated annealing. *Science*, Vol. 220, pp. 671–680.
- Seçer, M. (2009). Inelastic and large deformation analyses of plane trusses. *Technology*, Vol. 12(3), pp. 175-184.
- Toklu, Y. C. (2004). Nonlinear Analysis of Trusses through Energy Minimization. *Computers and Structures*, Vol. 82, pp.1581-1589.
- Toklu, Y.C., Bekdaş G. and Temur R. (2013). Analysis of Trusses by Total Potential Optimization Method Coupled with Harmony Search. *Structural Engineering and Mechanics*, Vol. 45(2), pp. 183-199.
- Toklu, Y.C. and Toklu, N. E. (2013). Analysis of structures by Total Potential Optimization using Meta-heuristic Algorithms (TPO/MA). In Siarry, P. *Heuristics: Theory and Applications*, Nova Science. Chapter 16. pp. 345-374.